

Thus, in a right triangle ABC (see Figure 9–7), we have the following relations:

$$1. \sin A = \frac{a}{c} = \cos B$$

$$2. \cos A = \frac{b}{c} = \sin B$$

$$3. \tan A = \frac{a}{b} = \cot B$$

$$4. \cot A = \frac{b}{a} = \tan B$$

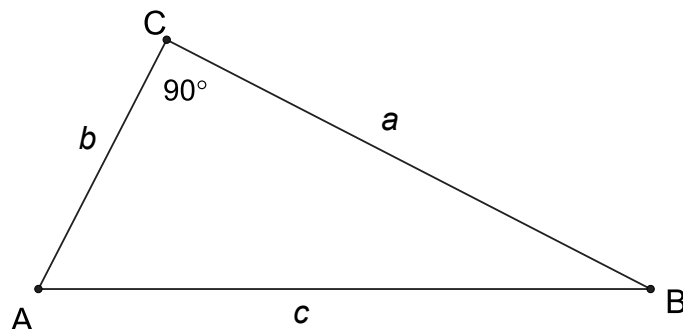


FIGURE 9–7

9.4 RELATIONS AMONG TRIGONOMETRIC FUNCTIONS

There are six basic relations among trigonometric functions. They are referred to as basic **identities**:

$$1. \tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$2. \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$3. \tan \alpha \cdot \cot \alpha = 1$$

$$4. \sin^2 \alpha + \cos^2 \alpha = 1$$

$$5. \tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}, \cos \alpha \neq 0$$

$$6. 1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha}, \sin \alpha \neq 0$$

Often $\frac{1}{\sin \alpha}$ is denoted by $\csc \alpha$ and $\frac{1}{\cos \alpha}$ by $\sec \alpha$.

Thus identities 5 and 6 can be written as:

$$7. \tan^2 \alpha + 1 = \sec^2 \alpha$$

$$8. 1 + \cot^2 \alpha = \csc^2 \alpha.$$

To prove these formulas for an acute angle α , let $\triangle ABC$ be a right triangle in C with the acute angle $\angle BAC = \alpha$. Let $AC = b$, $CB = a$, and $AB = c$, as shown in Figure 9–8.